THE CORRESPONDENCE BETWEEN THE FORMAL DOUBLE POWER SERIES AND TWO-DIMENSIONAL \(g\)-FRACTION WITH NONEQUIVALENT VARIABLES

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One of the methods of expanding the functions of two variables, given by the formal double power series, into the two-dimensional continued fractions is the construction of corresponding two-dimensional continued fractions [1].

We consider the two-dimensional \(g\)-fraction with nonequivalent variables

\[
\Phi_0(z_1) + \sum_{n=1}^{\infty} \frac{g_{0n}(1-g_{0,n-1})z_2}{\Phi_n(z_1)} \quad \text{for} \quad l \geq 0,
\]

where \(s_0 > 0\), \(g_{00} = 0\), \(0 < g_{kl} < 1\), \(k \geq 0\), \(l \geq 0\), \(k + l > 0\), \(z = (z_1, z_2) \in \mathbb{C}^2\), which is generalization of the continued \(g\)-fraction [2].

The correspondence between the two-dimensional \(g\)-fraction with nonequivalent variables (1) and the formal double power series

\[
\sum_{k,l=0}^{\infty} (-1)^{k+l}s_{kl}z_1^kz_2^l,
\]

where \(s_{kl} \in \mathbb{R}\), \(k \geq 0\), \(l \geq 0\), \(z \in \mathbb{C}^2\), means that the expansion of each \(n\)th approximant, \(n \geq 1\), into the formal double power series coincides with the given series for all homogeneous polynomials to the degree \(\nu_n - 1\) inclusively. The \(\nu_n\) is called the order of correspondence.

We prove the following theorem:

**Theorem 1.** For the two-dimensional \(g\)-fraction with nonequivalent variables (1) there exists the unique formal double power series of form (2) to which this fraction will correspond. The order of correspondence is \(\nu_n = n\).

The following theorem deals with the convergence of corresponding two-dimensional \(g\)-fraction with nonequivalent variables to formal double power series.

**Theorem 2.** The two-dimensional \(g\)-fraction with nonequivalent variables (1) converges in the domain \(Q = \{z \in \mathbb{C}^2 : |z_1| < 1/2, |z_2| < 1/2\}\) to function \(g(z)\) which is holomorphic in this domain. The sum of the formal double power series (2), which corresponds to the two-dimensional \(g\)-fraction with nonequivalent variables (1), has the same value as this fraction in the domain \(Q\).

**References**


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